Steady-State Marginalized Particle Filter for Attitude Estimation

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Outline

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Introduction

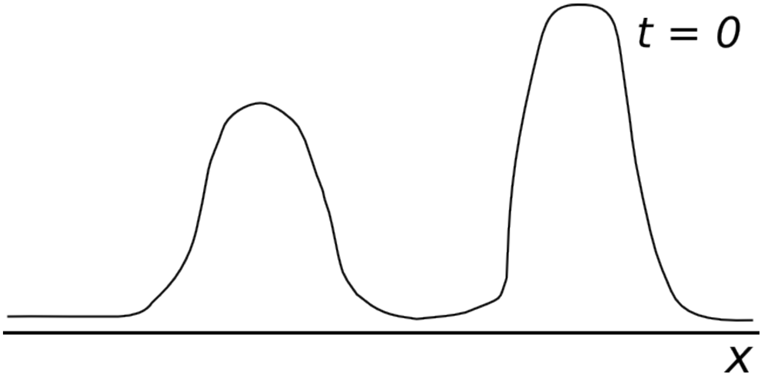
- Attitude estimation is *finding the best guess* to a spacecraft's orientation
- *Particle Filters* recently gained traction as a feasible estimation tool because of advances in computational power
- It also doesn't make any assumptions at all about the states' belief, in contrast to EKF and UKF, which have been traditionally used

Particle Filter: Big Idea

- Based on Bayes rule, particle filter is a Monte Carlo method to arrive at a best estimate for the state of a system

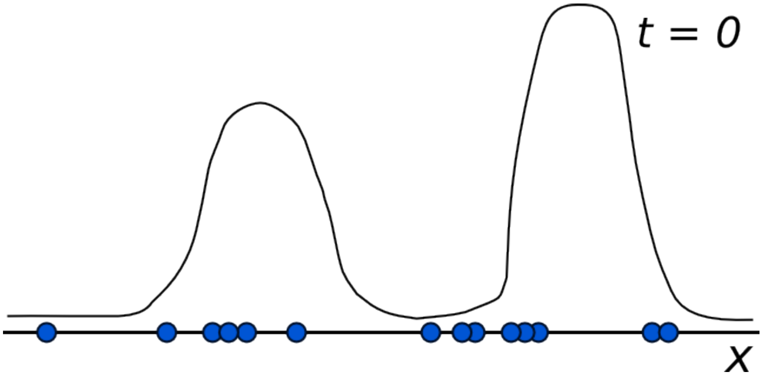
$$P(x_t|Y_t) = \eta P(y_t|x_t)P(x_t|Y_{t-1}) \tag{1}$$

Initialize



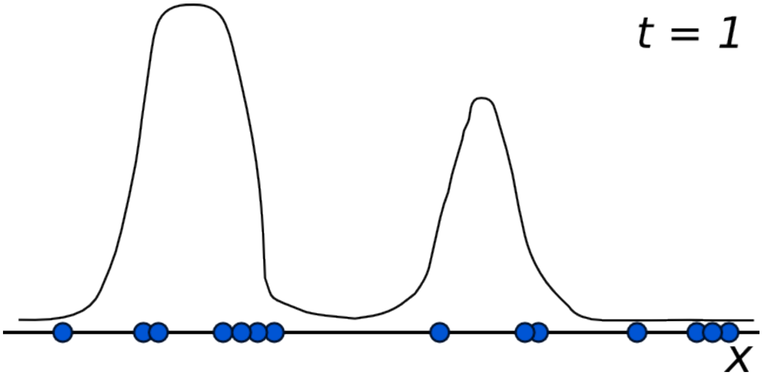
Some unknown, nonlinear state distribution

Initialize



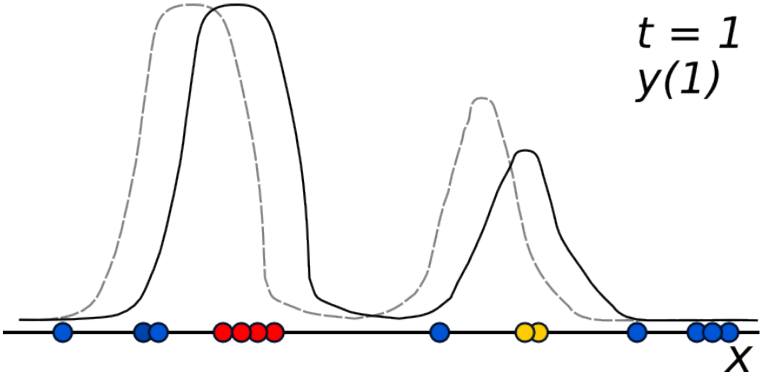
$$x_{0|-1}^{(i)} \sim P(x_0), \quad \forall i = 1, \ldots, N$$

Propagate



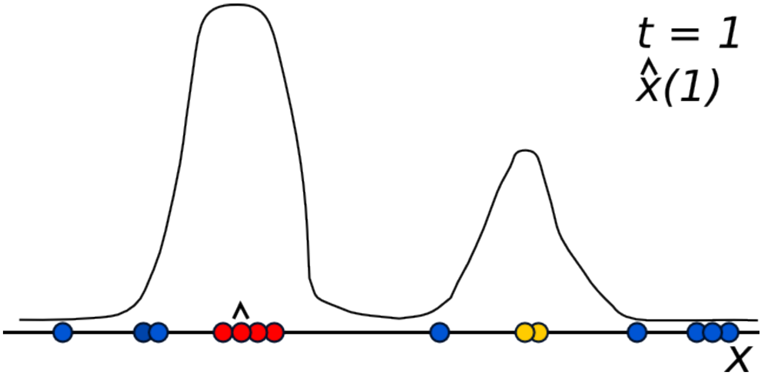
$$x_{t+1|t}^{(i)} \sim P(x_{t+1}|x_{t|t}^{(i)}, u_t)$$

Update



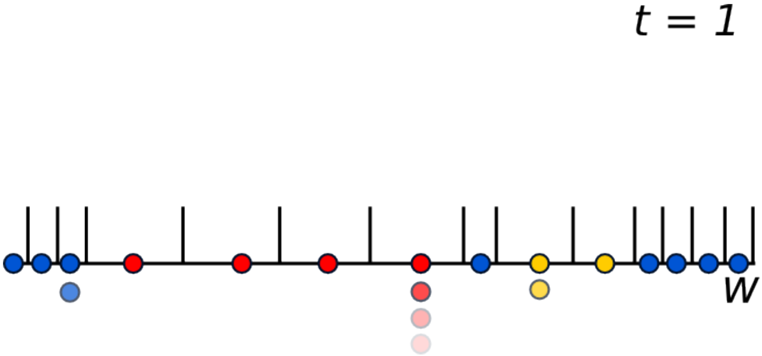
$$w_t^{(i)} \propto P(y_t | x_{t|t-1}^{(i)}) w_{t-1}^{(i)} \text{ and } \sum_{i=1}^{N} w_t^{(i)} = 1$$

Average



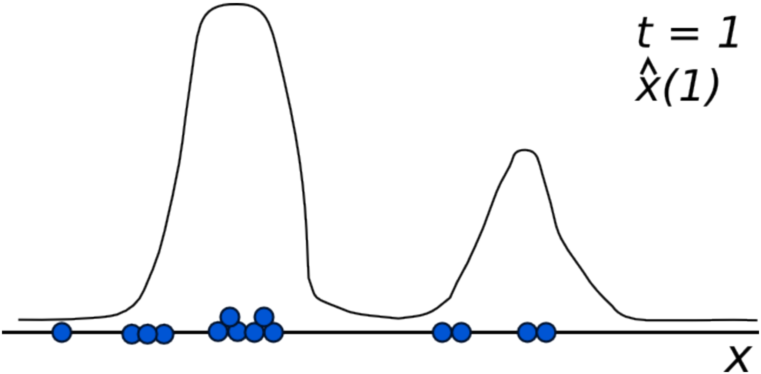
$$\hat{x}_{t|t} = \sum_{i=1}^{N} w_t^{(i)} x_{t|t-1}^{(i)}$$

Resample



$$Pr(x_{t|t}^{(i)} = x_{t|t-1}^{(j)}) = w_t^{(j)}$$

Resample



Reweight $w_t^{(i)} = 1/N$ and then repeat

Marginalized PF: Big Idea

- MPF is a natural extension of PF and takes advantage of the linear substructure hidden in the filter dynamics by marginalizing them and estimating them with a Kalman filter.

$$x_t = \begin{bmatrix} x_t^n \\ x_t^l \end{bmatrix} = \begin{bmatrix} q_t \\ \beta_t \end{bmatrix} \tag{2}$$

MPF Formulation

- It turns out that the dynamics attitude estimation can fit in the MPF framework!

$$
\begin{aligned}
x_{t+1}^n &= f_t^n(x_t^n) + A_t^n(x_t^n)x_t^l + G_t^n(x_t^n)w_t^n \\
x_{t+1}^l &= f_t^l(x_t^n) + A_t^l(x_t^n)x_t^l + G_t^l(x_t^n)w_t^l \\
y_t &= h_t(x_t^n) + C_t(x_t^n)x_t^l + e_t
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
q_{t+1} &= q_t + \frac{\Delta t}{2}\Xi(q_t)\tilde{\omega}_t - \frac{\Delta t}{2}\Xi(q_t)\beta_t - \frac{\Delta t}{2}\Xi(q_t)\zeta_t \\
\beta_{t+1} &= \beta_t + \Delta t \cdot \eta_t \\
y_i &= A(q)r_i + \nu_i
\end{aligned} \tag{4}
$$

[14] Schon, T., Gustafsson, F., and Nordlund, P.-J., 2005. Marginalized particle filters for mixed linear/nonlinear state-space models. *Signal Processing*, IEEE Transactions on, 53(7), pp. 22792289.
[5] Crassidis, J. L., and Junkins, J. L., 2011. *Optimal estimation of dynamic systems*. CRC press.

MPF Formulation

- We can match terms and find that:

$$
\begin{aligned}
f_t^n &= q_t + \frac{\Delta t}{2}\Xi(q_t)\tilde{\omega} \\
A_t^n &= G_t^n = -\frac{\Delta t}{2}\Xi(q_t) \\
A_t^l &= G_t^l/\Delta t = I_{3\times 3} \\
h_t &= \begin{bmatrix} A(q_t)r_1 \\ A(q_t)r_2 \\ \vdots \\ A(q_t)r_M \end{bmatrix} \\
C_t &= 0_{3M\times 3} \quad f_t^l = 0_{3\times 1}
\end{aligned}
\tag{5}
$$

SS MPF Formulation

- We can now run the PF on the nonlinear states and the KF on the linear states to get our state estimates
- If each linear particle is subject to the same initial covariance conditions, then the N linear particles can be propagated by one set of KF equations.
- Implication: We would reduce our problem size because each linear particle can be initialized with the same Ricatti equation

SS MPF Formulation

- If the right conditions exists, Kalman theory dictates that there exists a steady state covariance P_∞, and subsequently, a steady state Kalman gain, K_∞

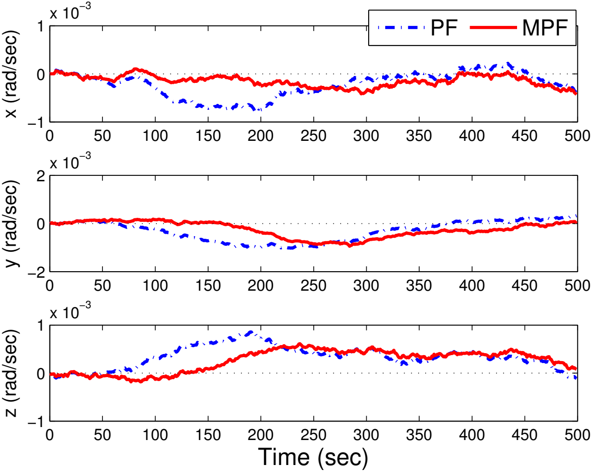
$$\begin{aligned} P_\infty &= P_\infty + \Delta t^2 \bar{Q}_2 - P_\infty (P_\infty + \bar{Q}_1)^{-1} P_\infty \\ K_\infty &= P_\infty (P_\infty + \bar{Q}_1)^{-1} \end{aligned} \tag{6}$$

- Implication: No more propagating each particle's covariance through the MPF and you can solve for the gains offline

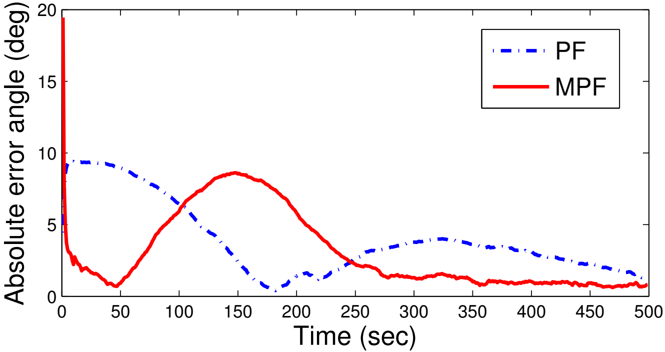
PF vs SS MPF

	PF	**Steady-state MPF**
Initialization	Initialize particles $q_0^{(i)}, \beta_0^{(i)}$	Initialize quaternion particles $q_0^{(i)}$ $\beta_0^{(i)} = \bar{\beta}_0$ Solve the Riccati equation for P_∞
Update	$w_t^{(i)} \propto P(y_t \| x_{t\|t-1}^{(i)}) w_{t-1}^{(i)}$ and $\sum_{i=1}^N w_t^{(i)} = 1$	same as PF
Estimate	$\hat{x}_{t\|t} = \sum_{i=1}^N w_t^{(i)} x_{t\|t-1}^{(i)}$	same as PF
Resampling	$Pr(x_{t\|t}^{(i)} = x_{t\|t-1}^{(j)}) = w_t^{(j)}$	same as PF
Propagation	$\omega_t^{(i)} = \tilde{\omega}_t - \beta_t^{(i)} - \eta^{(i)}$ $q_{t+1}^{(i)} = q_t^{(i)} + \frac{\Delta t}{2} \Xi(q_t^{(i)}) \omega_t^{(i)}$ $\beta_{t+1}^{(i)} = \beta_t^{(i)} + \Delta t \zeta^{(i)}$ $\eta^{(i)} \sim \mathcal{N}(0_{3\times1}, \bar{Q}_2), \zeta^{(i)} \sim \mathcal{N}(0_{3\times1}, \bar{Q}_1)$	$\omega_t^{(i)} = \tilde{\omega}_t - \beta_t^{(i)} - \upsilon^{(i)}$ $q_{t+1}^{(i)} = q_t^{(i)} + \frac{\Delta t}{2} \Xi(q_t^{(i)}) \omega_t^{(i)}$ $\beta_{t+1}^{(i)} = \beta_t^{(i)} + K_\infty \upsilon^{(i)}$ $\upsilon^{(i)} \sim \mathcal{N}(0_{3\times1}, P_\infty + \bar{Q}_1)$

Simulation Results



Simulation Results



Conclusion

- The attitude estimation equations can fit in the MPF framework
- We showed that there exists a steady state PF that can offer additional advantages in relieving computational burden
- We were able to show that MPF can offer improvements over the PF and EKF in estimating, especially for large initial error covariance, the attitude

Conclusion

Thank you for your time!

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