Using Trajectory Optimization for Free Space Optical Communication

Masters Presentation
Dennis Wai
Professor Masayoshi Tomizuka
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Outline

- Introduction and Overview
- Trajectory Optimization
- Dynamics
- Constraints
- Time Varying LQR
- Simulation
- Conclusion

Overview

- Free Space Optical (FSO) communication is the use of lasers instead of radio waves to exchange information between two terminal points

Artist Rendering of FSO

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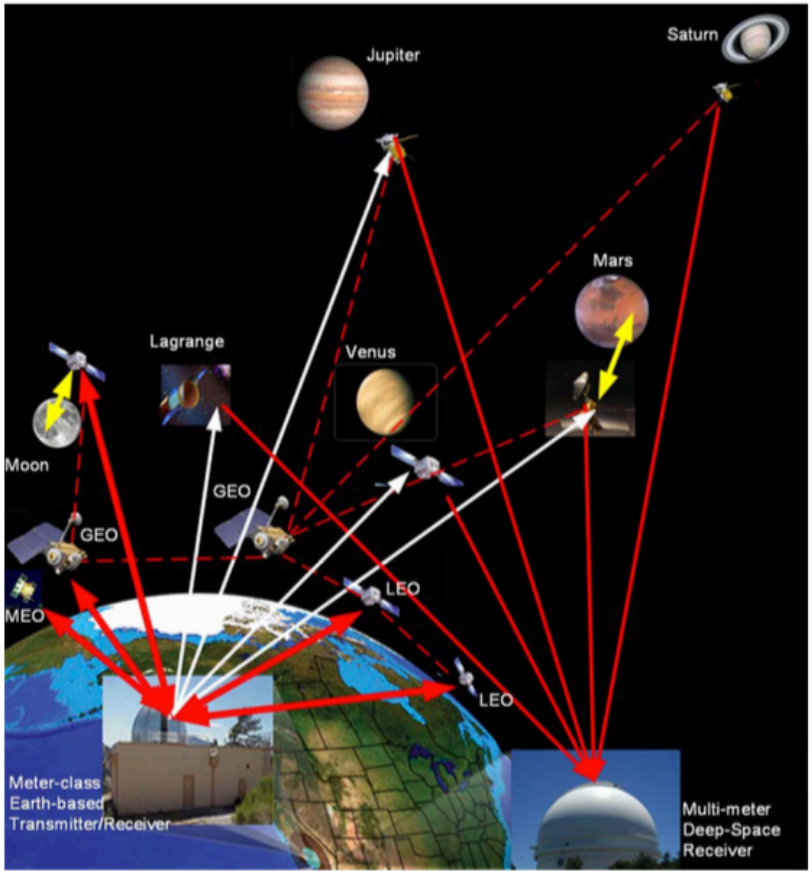
System Overview

- Two terminals: emitter and receiver
  - Earth to LEO
  - LEO to LEO
  - Earth to GEO



Overview of types of satellite links

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Motivation

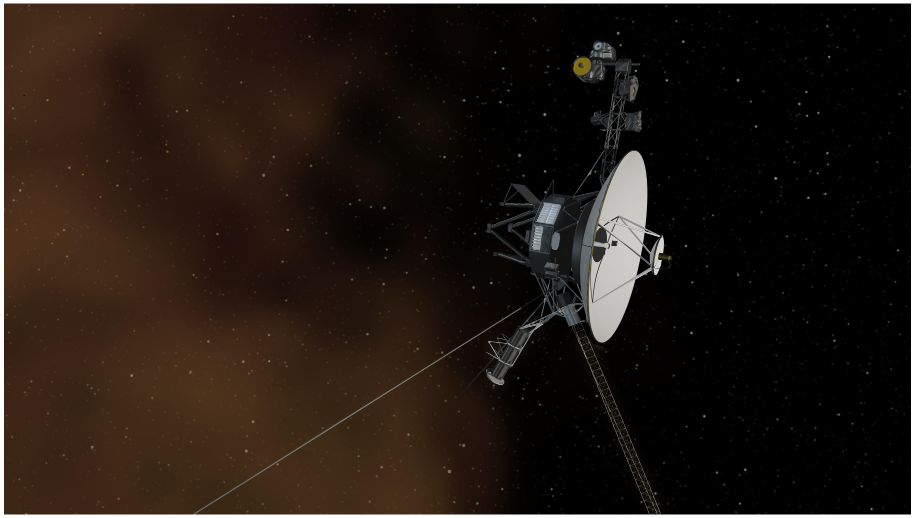
- Simplified craft design
  - Power, mass, volume savings
- Ease of spacecraft packaging

Link	Optical	RF
GEO-LEO		
Antenna Diameter	10.2 cm (1.0)	2.2 m (21.6)
Mass	65.3 kg (1.0)	152.8 kg (2.3)
Power	93.8 W (1.0)	213.9 W (2.3)
GEO-GEO		
Antenna Diameter	13.5 cm (1.0)	2.1 m (15.6)
Mass	86.4 kg (1.0)	145.8 kg (1.7)
Power	124.2 W (1.0)	204.2 W (1.6)
LEO-LEO		
Antenna Diameter	3.6 cm (1.0)	0.8 m (22.2)
Mass	23.0 kg (1.0)	55.6 kg (2.4)
Power	33.1 W (1.0)	77.8 W (2.3)

Table I
COMPARISON OF POWER AND MASS FOR GEOSTATIONARY EARTH ORBIT (GEO) AND LOW EARTH ORBIT (LEO) LINKS USING OPTICAL AND RF COMMUNICATION SYSTEMS (VALUES IN PARENTHESES ARE NORMALIZED TO THE OPTICAL PARAMETERS)

Motivation

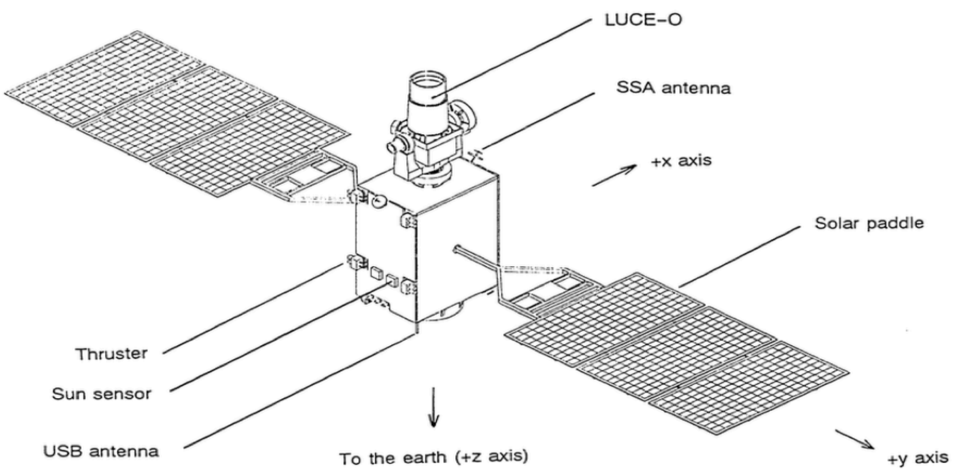
- Faster data throughput
- Transmission rate improvements from kbps to Gbps



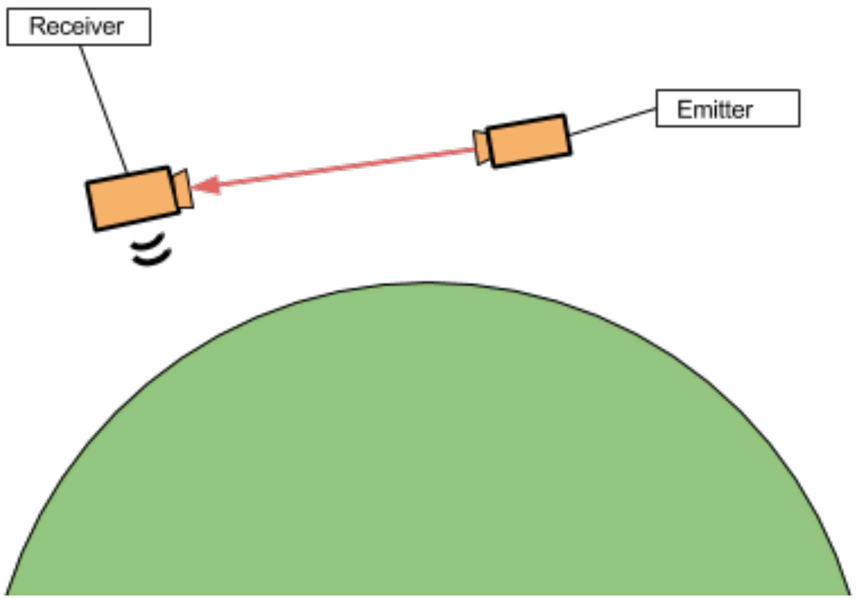
Rendering of *Voyager* space probe

System Overview

- Turret-mounted laser
  - Assumption was made in this work that turret rigidly attached to satellite
- Nadir point
  - For communications with Earth, this nadir point needs to point towards Earth



Free Space Optical Communication



Simplified Diagram for Free Space Optical Communication

- Two terminals, a receiver and emitter establish a line of sight with a priori information
- Both receiver and emitter maintain line of sight during a transmission with each other

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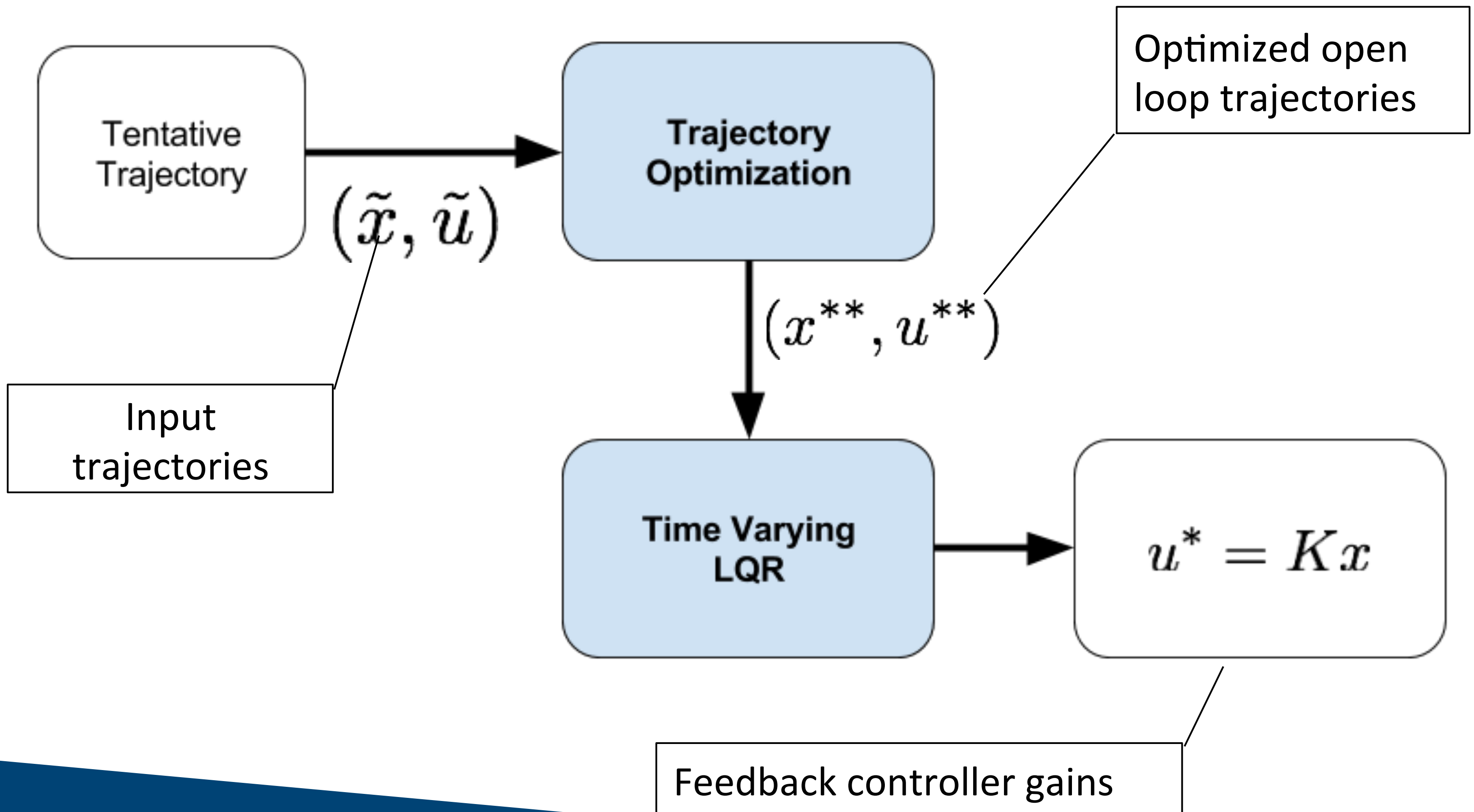
Limitations

- Receiver orbital trajectory over a horizon isn’t leveraged in controller prescription
- Constraints over control inputs are ignored
- Controller prescription also does not extend to multi-agent scenarios

Trajectory Optimization



Trajectory Optimization

- Trajectory optimization as a framework
  - Include input constraints
  - Include nonlinear constraints
  - Leverage the receiver's known trajectory
  - Prescribe control of both crafts

$$
\begin{array}{ll}
\min\limits_{x,u} & \mathcal{T}(x, u, \tilde{x}, \tilde{u}) \\
s.t & \text{initial state} \\
 & \text{satellite dynamics} \\
 & u \in \mathcal{U} \\
 & \text{nadir points to Earth} \\
 & \text{crafts maintains pointing lock}
\end{array}
$$

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Trajectory Optimization

- Solves a nonlinear problem by iteratively solving convex approximations of the problem
  - Guesses are conservatively made and enforced via a box constraint
  - Only guesses that are representative of improvements of the original problem are used for the next iteration

Equations of Motion

- Orbital Dynamics

$$\ddot{r} - \frac{\hat{h}^2}{m^2 r^3} = -\frac{GMm}{r^2}$$
$$m\dot{r}^2\dot{\sigma} = 0$$

- Non-dimensionalized dynamics

$$\ddot{\rho} = \frac{1}{\rho^3} - \frac{1}{\rho^2}$$
$$\dot{\alpha} = \frac{1}{\rho^2}$$

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Equations of Motion

- Rotational Dynamics

$$\tau = \mathbf{J}\alpha + \omega \times \mathbf{J}\omega$$

$$\mathbf{J} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\lambda_1 \ddot{\psi} = (\lambda_2 - \lambda_3)\dot{\phi}\dot{\theta} + u_1$$

$$\lambda_2 \ddot{\theta} = (\lambda_3 - \lambda_1)\dot{\psi}\dot{\phi} + u_2$$

$$\lambda_3 \ddot{\phi} = (\lambda_1 - \lambda_2)\dot{\psi}\dot{\theta} + u_3$$

Equations of Motion

- State Definition for one satellite

$$x_t = \left[\rho \; \dot{\rho} \; \alpha \; \psi \; \dot{\psi} \; \theta \; \dot{\theta} \; \phi \; \dot{\phi}\right]$$

$$u_t = \left[u_1 \; u_2 \; u_3\right]$$

$$\dot{x}_t = \begin{bmatrix} \dot{\rho} \\ \frac{1}{\rho^3} - \frac{1}{\rho^2} \\ \frac{1}{\rho^2} \\ \dot{\psi} \\ \frac{1}{\lambda_1}((\lambda_2 - \lambda_3)\dot{\phi}\dot{\theta} + u_1) \\ \dot{\theta} \\ \frac{1}{\lambda_2}((\lambda_3 - \lambda_1)\dot{\psi}\dot{\phi} + u_2) \\ \dot{\phi} \\ \frac{1}{\lambda_3}((\lambda_1 - \lambda_2)\dot{\psi}\dot{\theta} + u_3) \end{bmatrix}$$

Pointing Constraint

- First observe that

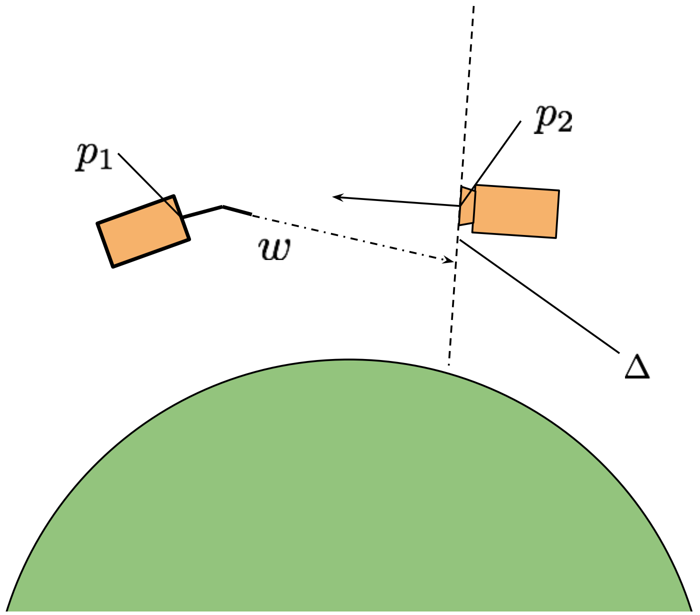
$$p_1 + \gamma w = \alpha v_1 + \beta v_2 + p_2$$

- I solved the matrix equation for coefficients

$$\Delta = \|p_1 + \gamma w - p_2\|_2$$

$$\text{err} = \frac{\Delta}{\eta}$$

$$\eta \geq 0$$



Simplified Diagram to formulate pointing constraint

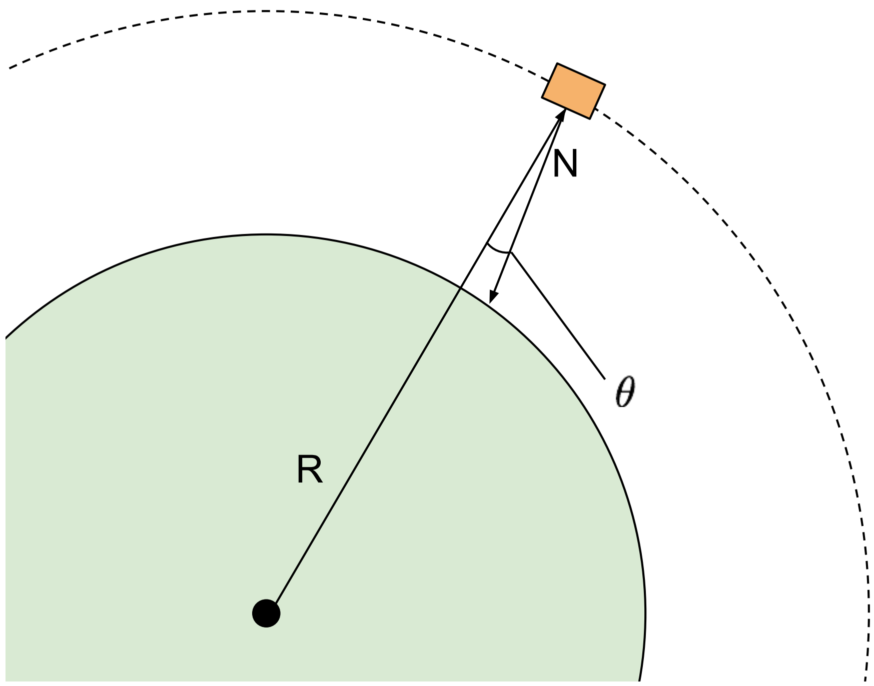
Nadir Constraint

- I observed that

$$\mathbf{R} \cdot \mathbf{N} = \|\mathbf{R}\| \|\mathbf{N}\| \cos\theta$$

- I normalized the vectors and formulated it as a constraint

$$g(x) \leq 0$$
$$\mathbf{R}(x) \cdot \mathbf{N}(x) - \cos\theta \leq 0$$
$$\delta := \cos\theta \leq 0$$



Simplified Diagram to formulate nadir constraint

Trajectory Optimization

LQ Cost

$$\min_{\tilde{x},\tilde{u}} \quad \sum_{t=1}^{T}(x_t - \tilde{x}_t)^T Q(x_t - \tilde{x}_t) + \sum_{t=1}^{T}(u_t - \tilde{u}_t)^T R(u_t - \tilde{u}_t)$$

$$s.t. \quad \tilde{x_0} = x_{init}$$

$$\forall t \in 1 \ldots T \ : \ x_{t+1} = f(x_t, u_t)$$

Dynamics

$$\forall t \in 1 \ldots T \ : u_t \in U$$

Input constraints

$$\forall t \in 1 \ldots T \ : \mathbf{R_s} \cdot \mathbf{N} + \delta < 0, \quad \delta < 0$$

Ex: nadir constraint

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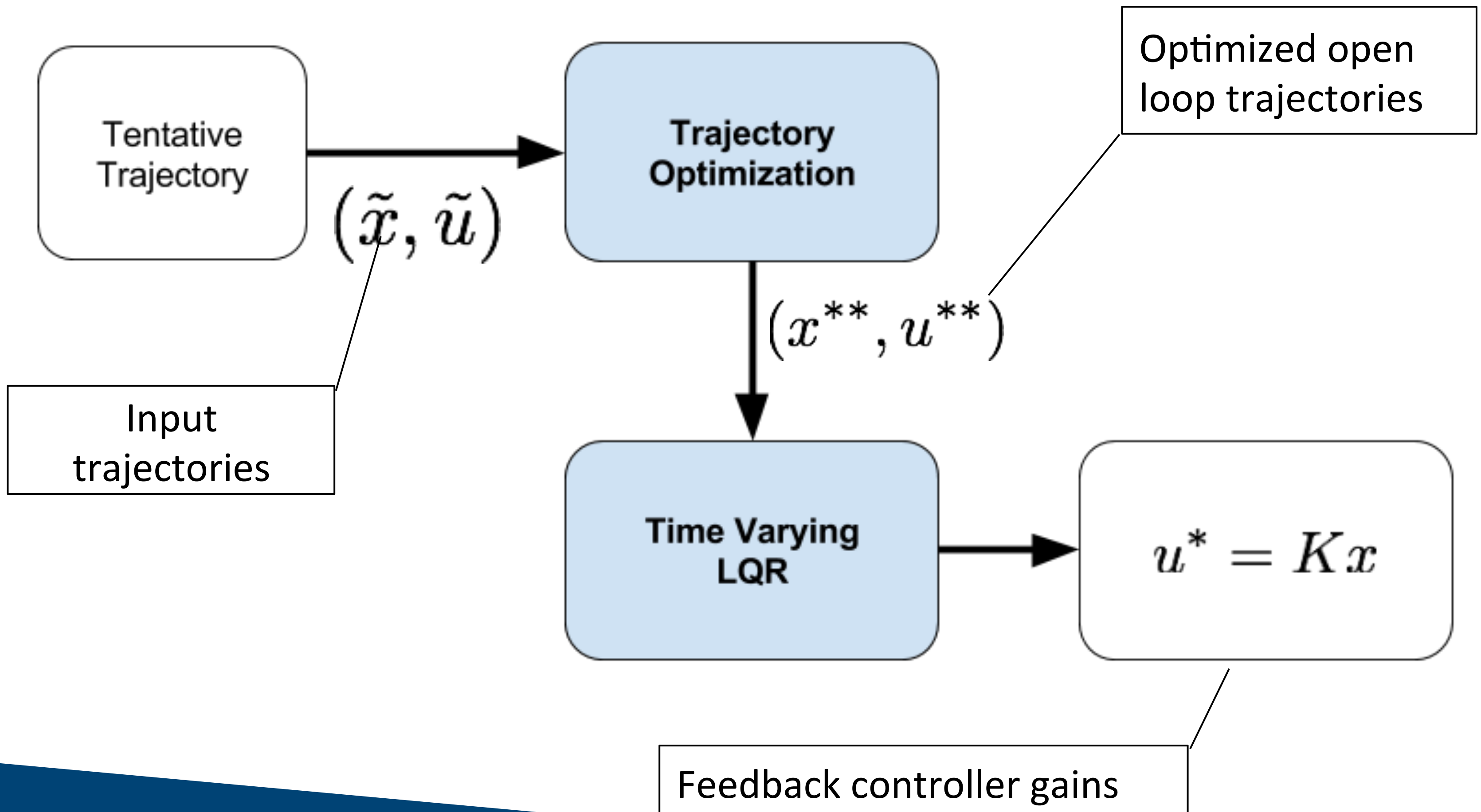
Trajectory Optimization



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Trajectory Generation

- Given a state representation for a satellite, we can create a trajectory for that satellite

$$\tilde{x} = \begin{bmatrix} x_1 \; x_2 \; \ldots \; x_T \end{bmatrix}$$
$$\tilde{u} = \begin{bmatrix} u_1 \; u_2 \; \ldots \; u_T \end{bmatrix}$$

- One example of a tentative trajectory would be satellite in orbit with zero control input

- If we have two satellites, we would stack one satellite's trajectory with another

Time Varying LQR

- Trajectory optimization will provide an OL trajectory

$$\text{traj. opt} \rightarrow x_t^{**}, u_t^{**}$$
$$x_{t+1}^{**} = f(x_t^{**}, u_t^{**})$$

- I chose to derive a time varying linear system on which we can run LQR

$$x_{t+1} - x_{t+1}^{**} = \frac{\partial f}{\partial x}(x_t - x_t^{**}) + \frac{\partial f}{\partial u}(u_t - u_t^{**})$$
$$x_{t+1} - x_{t+1}^{**} = A_t(x_t - x_t^{**}) + B_t(u_t - u_t^{**})$$

Time Varying LQR

- Optimal gain feedback backwards computation law
- *i* is the i-th step from end of horizon

$$P_0 = Q_0$$

$$K_i = -(R_{H-i} + B_{H-i}^T P_{i-1} B_{H-i})^{-1} B_{H-i}^T P_{i-1} A_{H-i}$$

$$P_i = Q_{H-i} + K_i^T R_{H-i} K_i + (A_{H-i} + B_{H-i} K_i)^T P_{i-1} (A_{H-i} + B_{H-i} K_i)$$

$$u_{H-i} - u_{H-i}^* = K_i (x_{H-i} - x_{H-i}^*)$$

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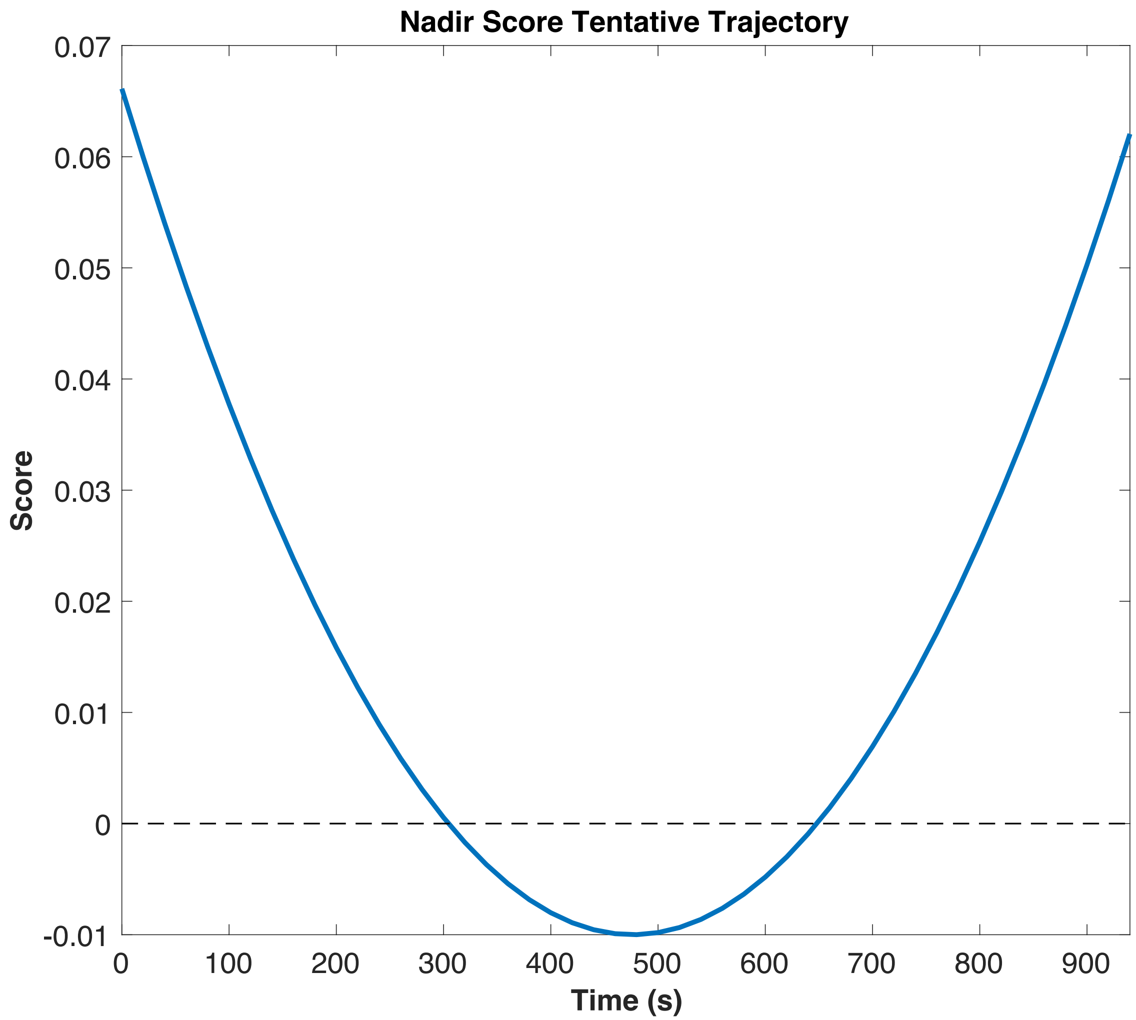
Nadir Constraint

- Plot of Nadir Constraint Score of the tentative trajectory
- Negative score is satisfactory
- Polar orbit of 45° inclination



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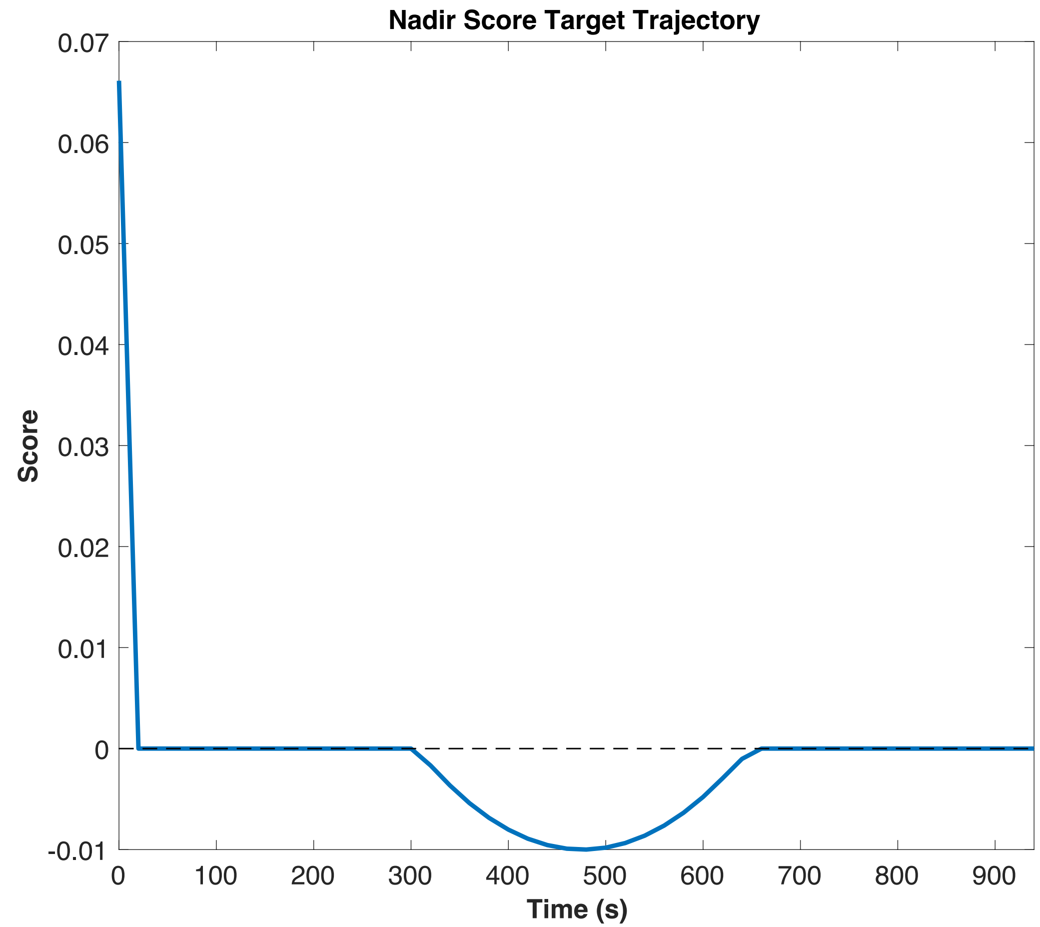
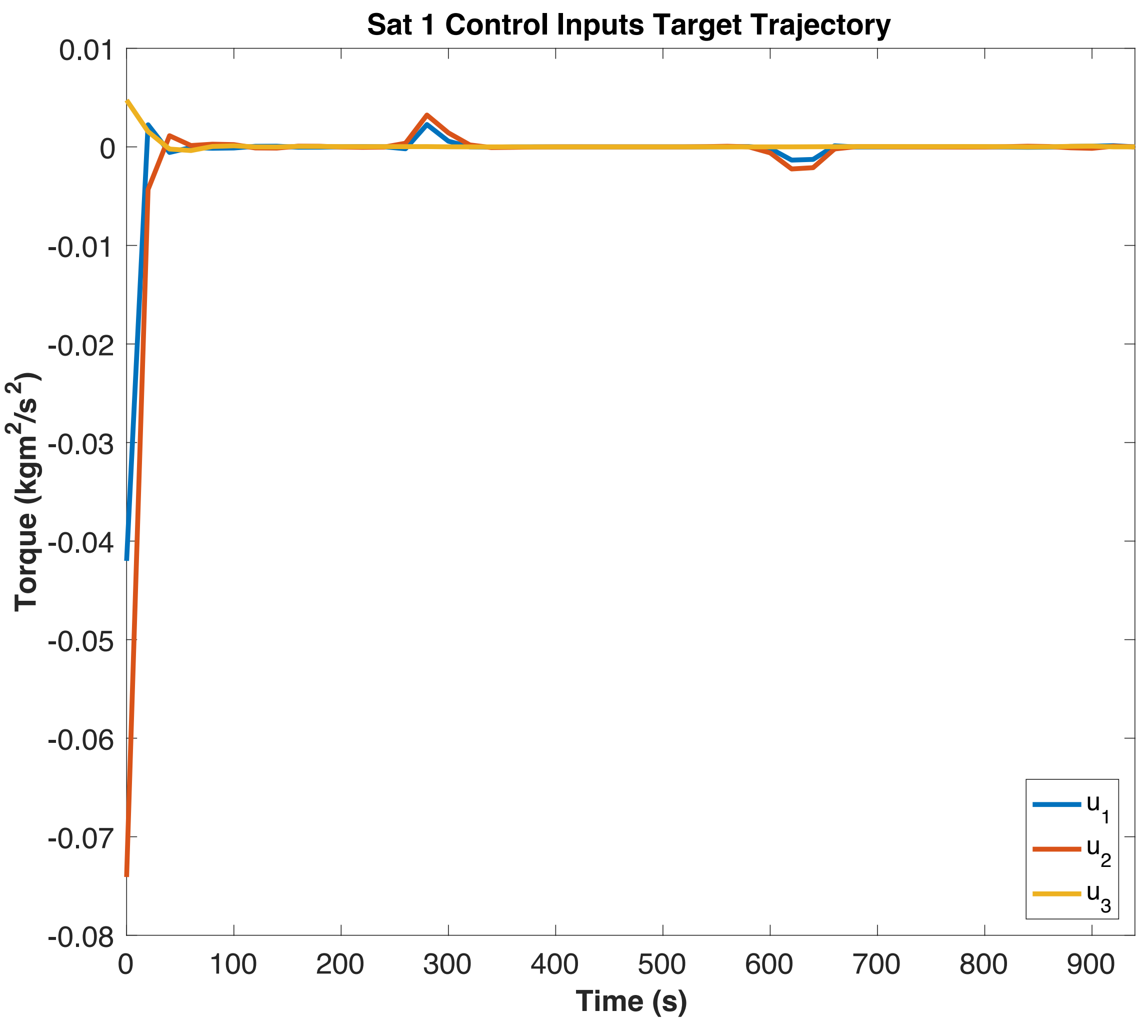
Nadir Constraint

- Plot of Nadir Constraint Score of the target trajectory



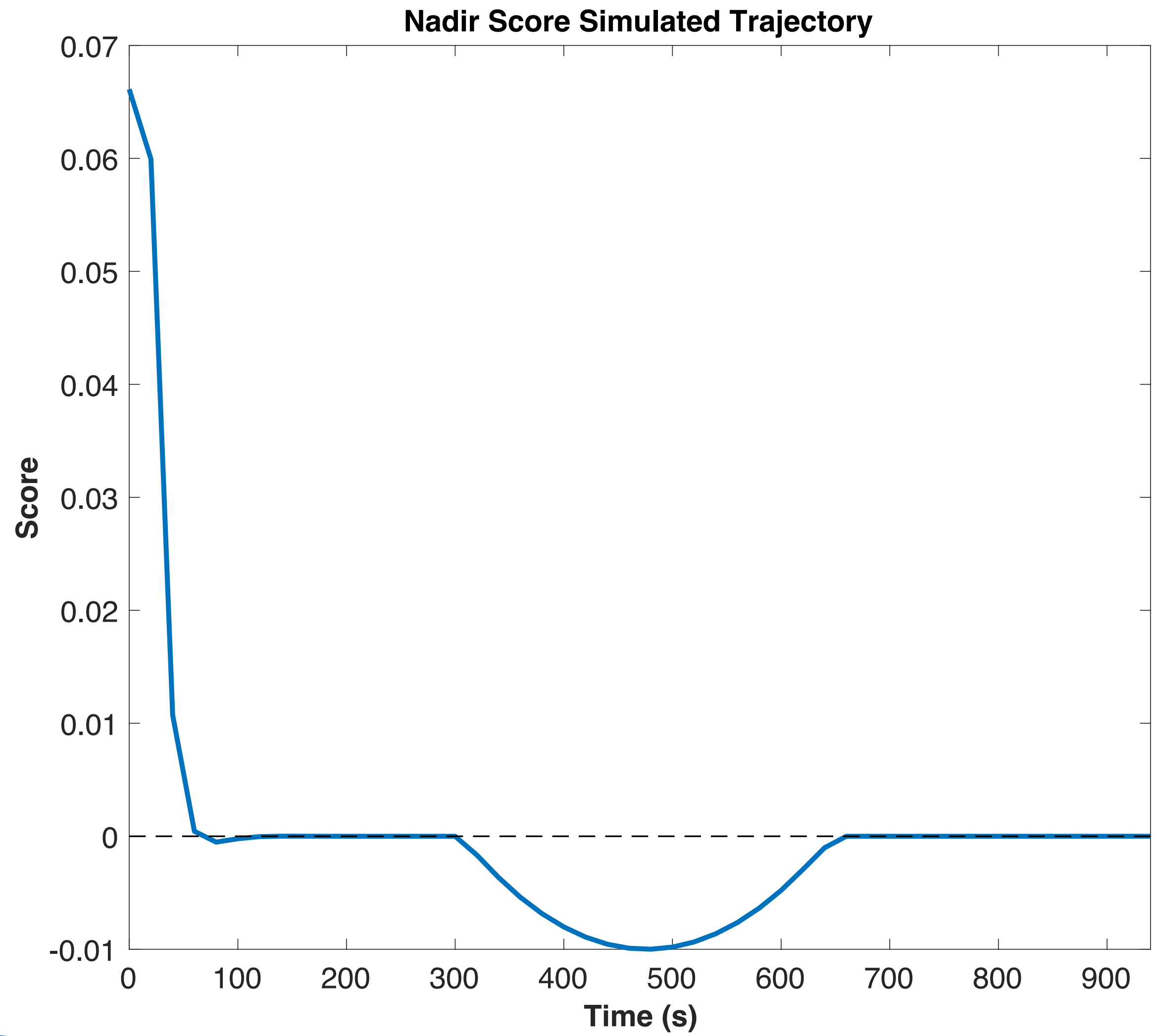
Nadir Constraint

- Plot of control inputs for the emitter satellite in the target trajectory



Nadir Constraint

- Plot of Nadir Constraint Score of the simulated trajectory



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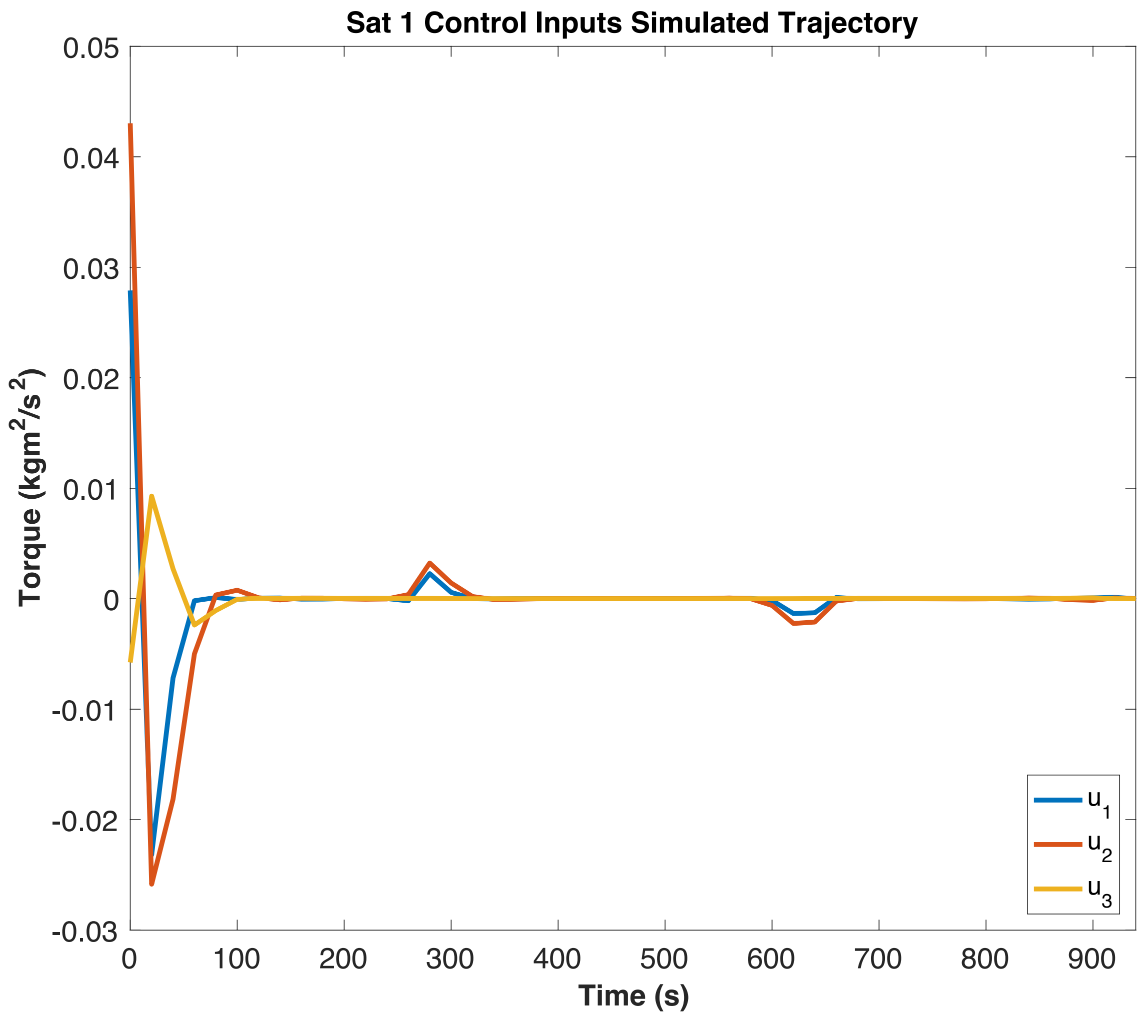
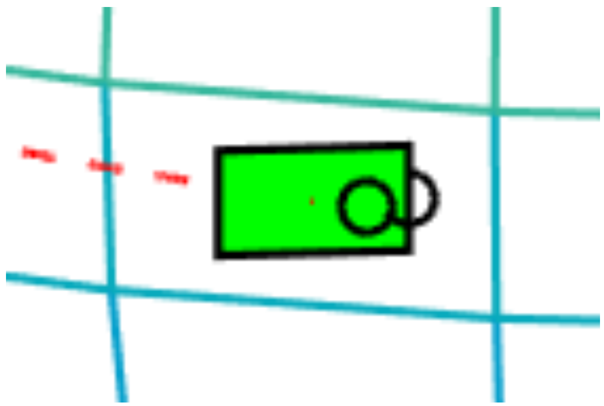
Nadir Constraint

- Plot of control inputs for the emitter satellite in the simulated trajectory



Optimized Orbit for Nadir Constraint



Satellite is represented by a black rectangle and the circle represents the nadir point

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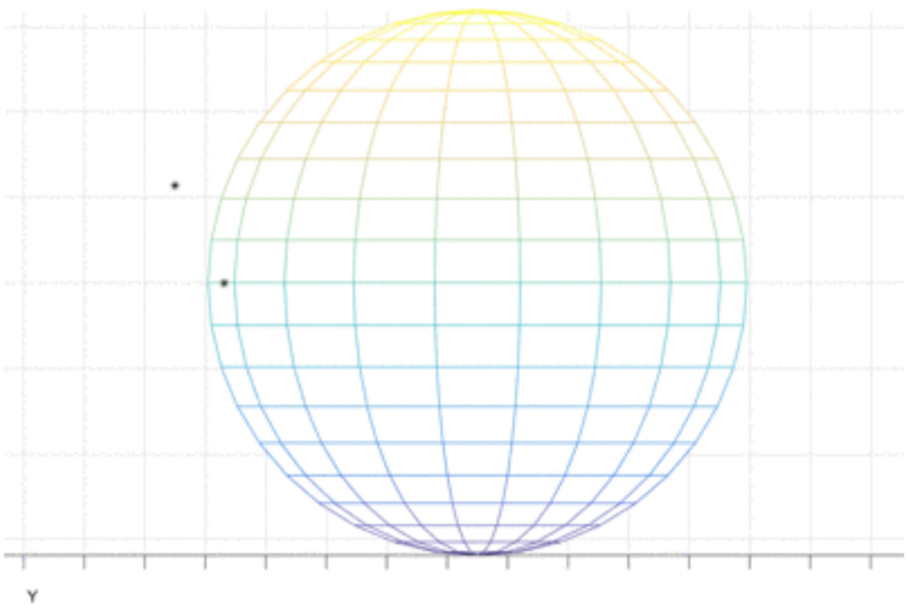
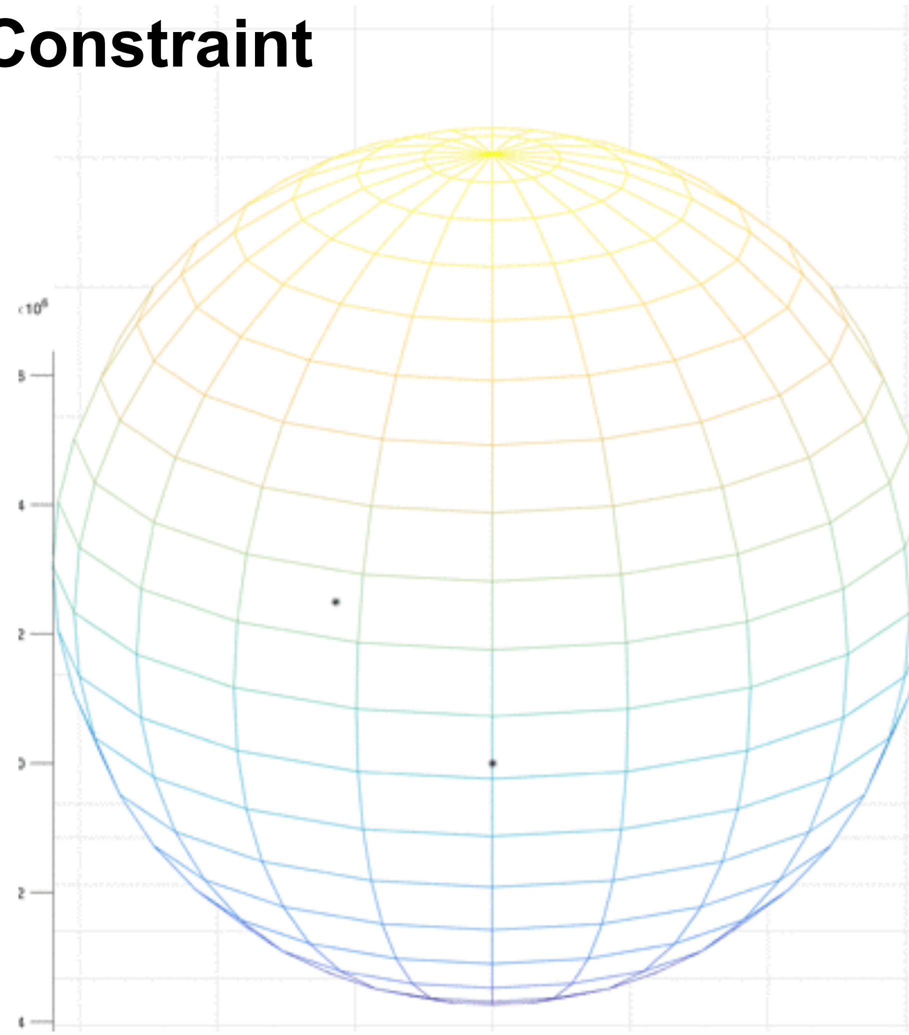
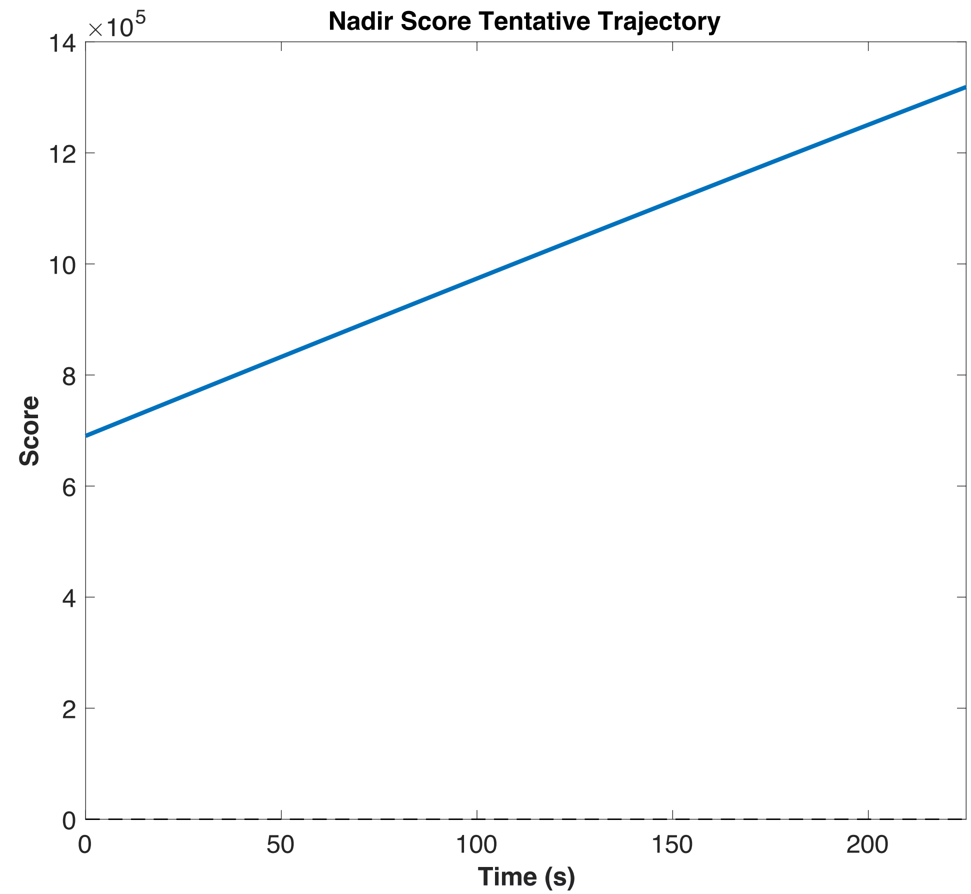
Optimized Orbit for Nadir Constraint





Pointing Constraint – 2500km

- Plot of Pointing Constraint Score of the tentative trajectory



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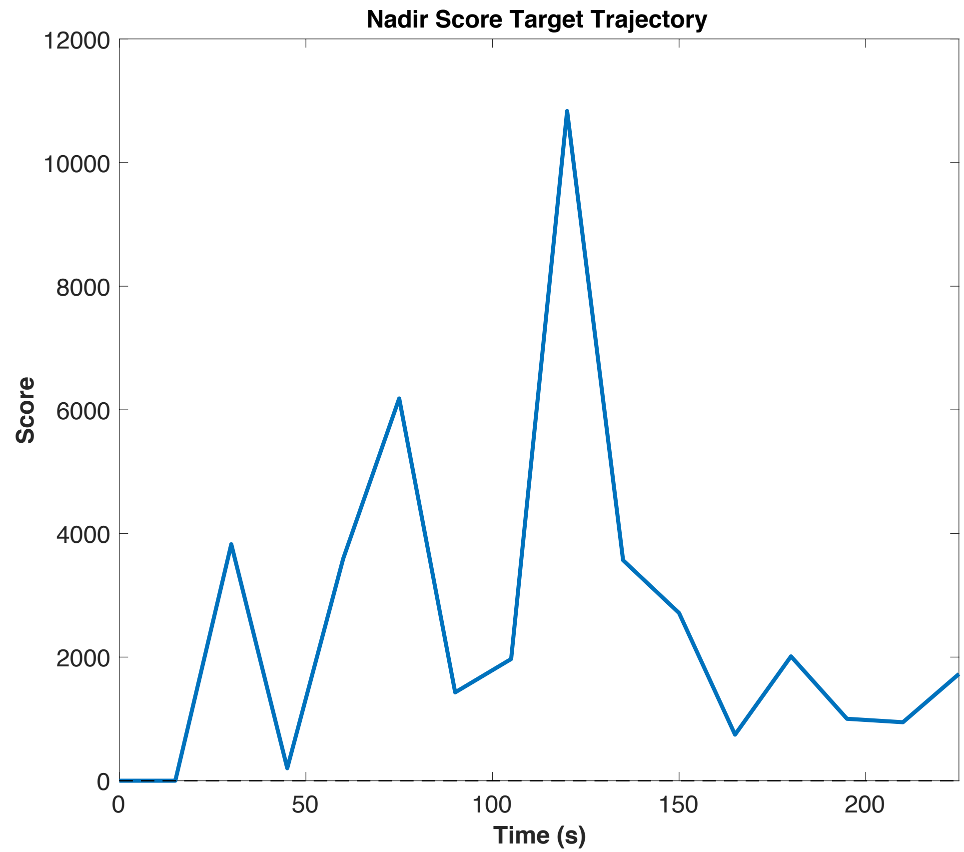
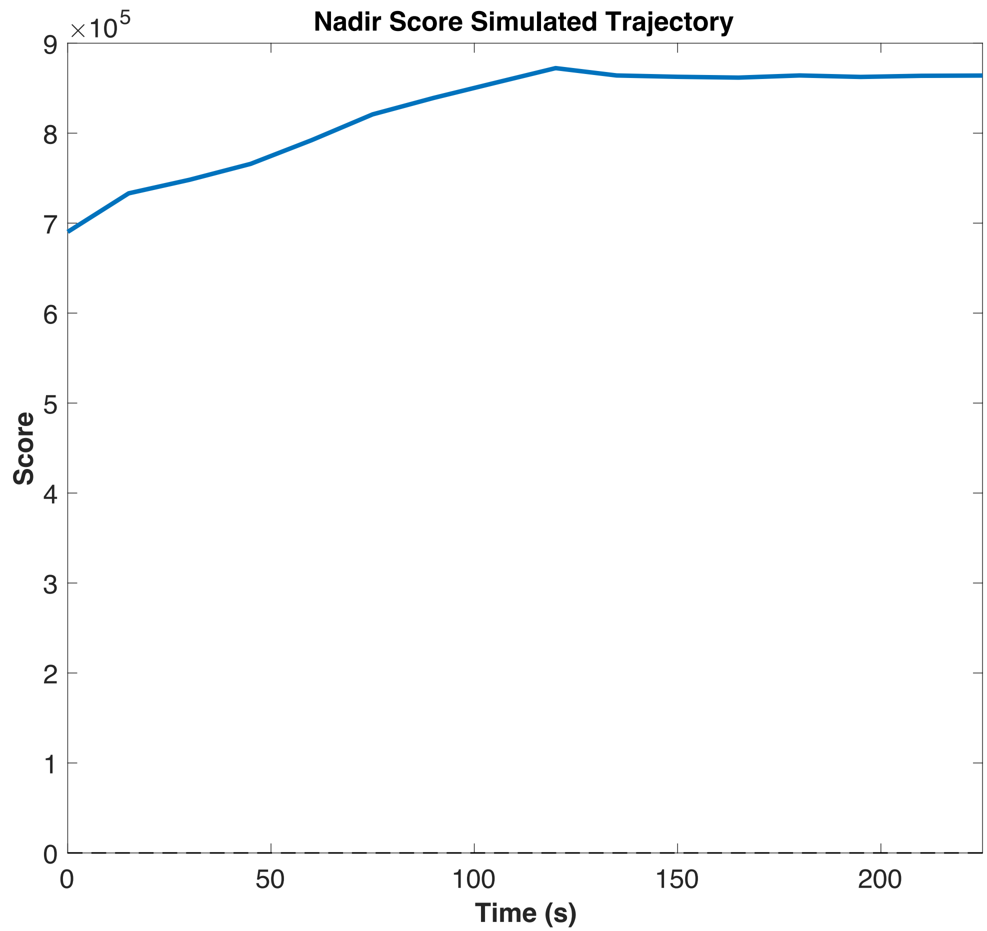
Pointing Constraint – 2500 km

- Plot of Pointing Constraint Score of the target trajectory



Pointing Constraint – 2500km

- Plot of Pointing Constraint Score of the simulated trajectory



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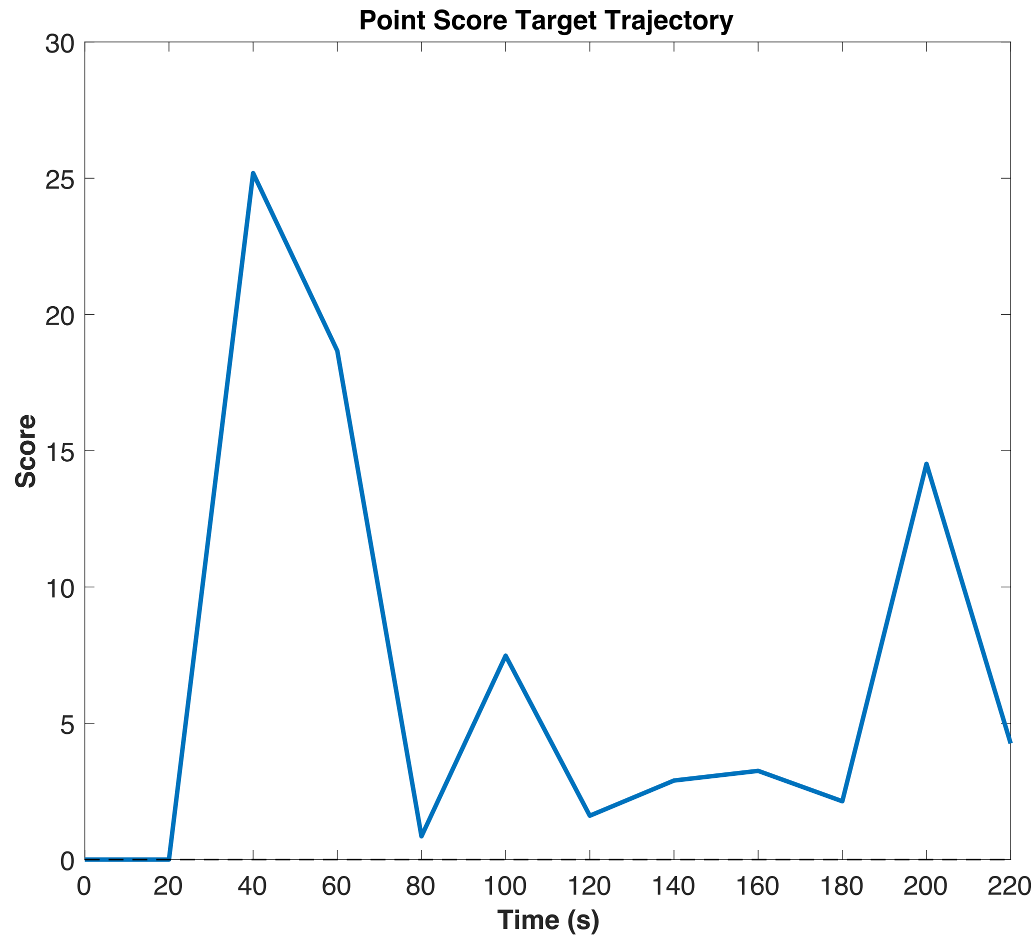
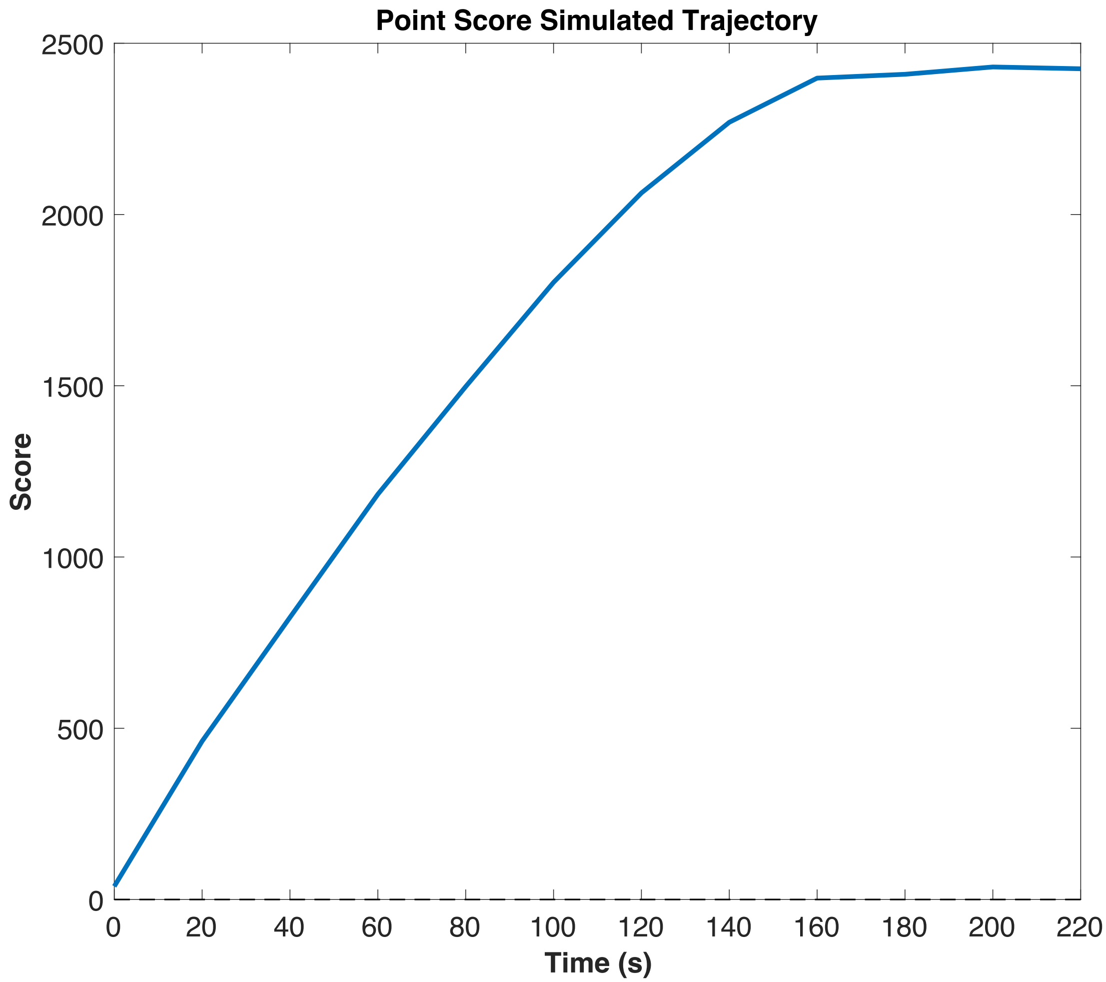
Pointing Constraint – 120km

- Plot of Pointing Constraint Score of the target trajectory



Pointing Constraint – 120km

- Plot of Pointing Constraint Score of the simulated trajectory



Trajectory Optimization Conclusion

- I expressed the problem of free space optical communication into a trajectory optimization problem in order to generate optimized trajectories that obeyed prescribed constraints
- I formulated performance criteria into constraints that can be plugged into trajectory optimization
- I used a time-varying LQR method on the optimized trajectories to generate a set of feedback controllers to implement online in simulation

Thank you for your time!

Questions?

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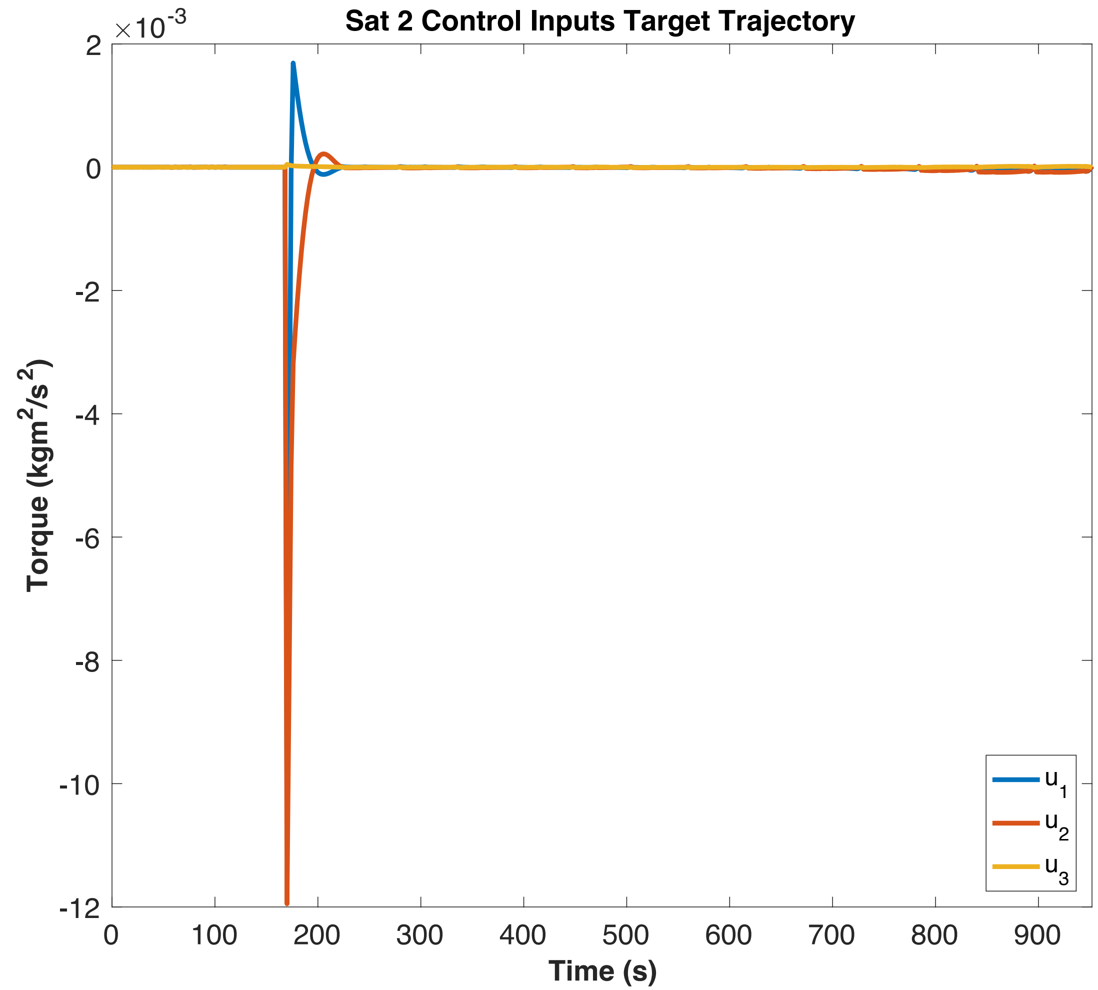
Back Up

Nadir Constraint

- Target, sat2



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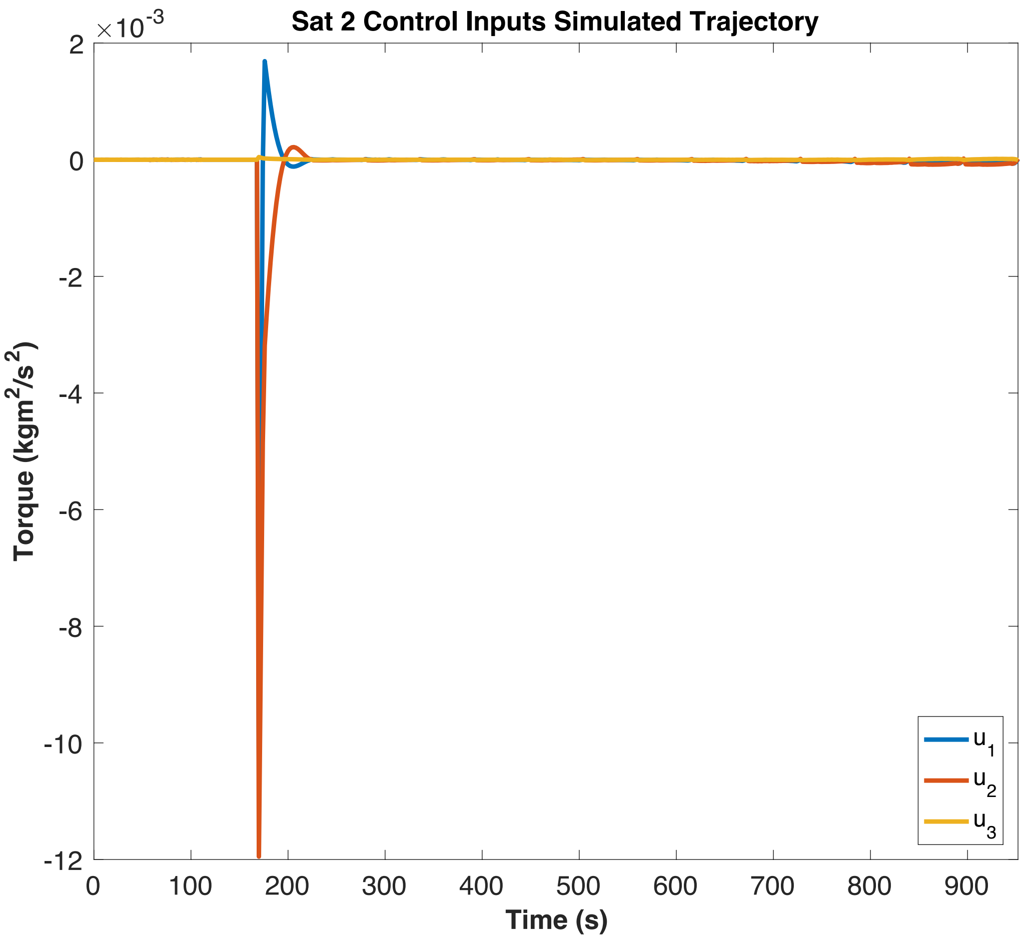
Nadir Constraint

- Simulated orbit



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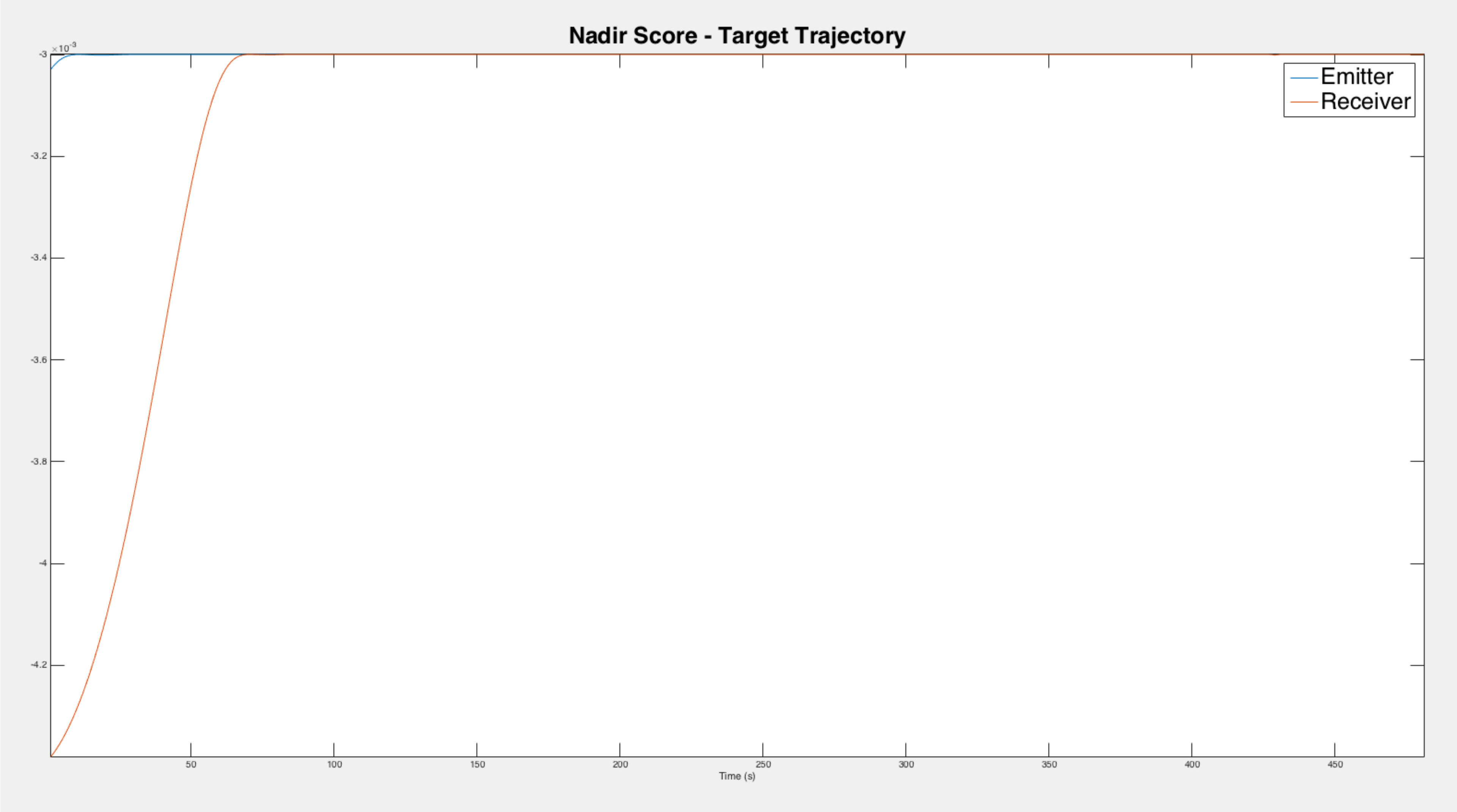
Nadir Constraint



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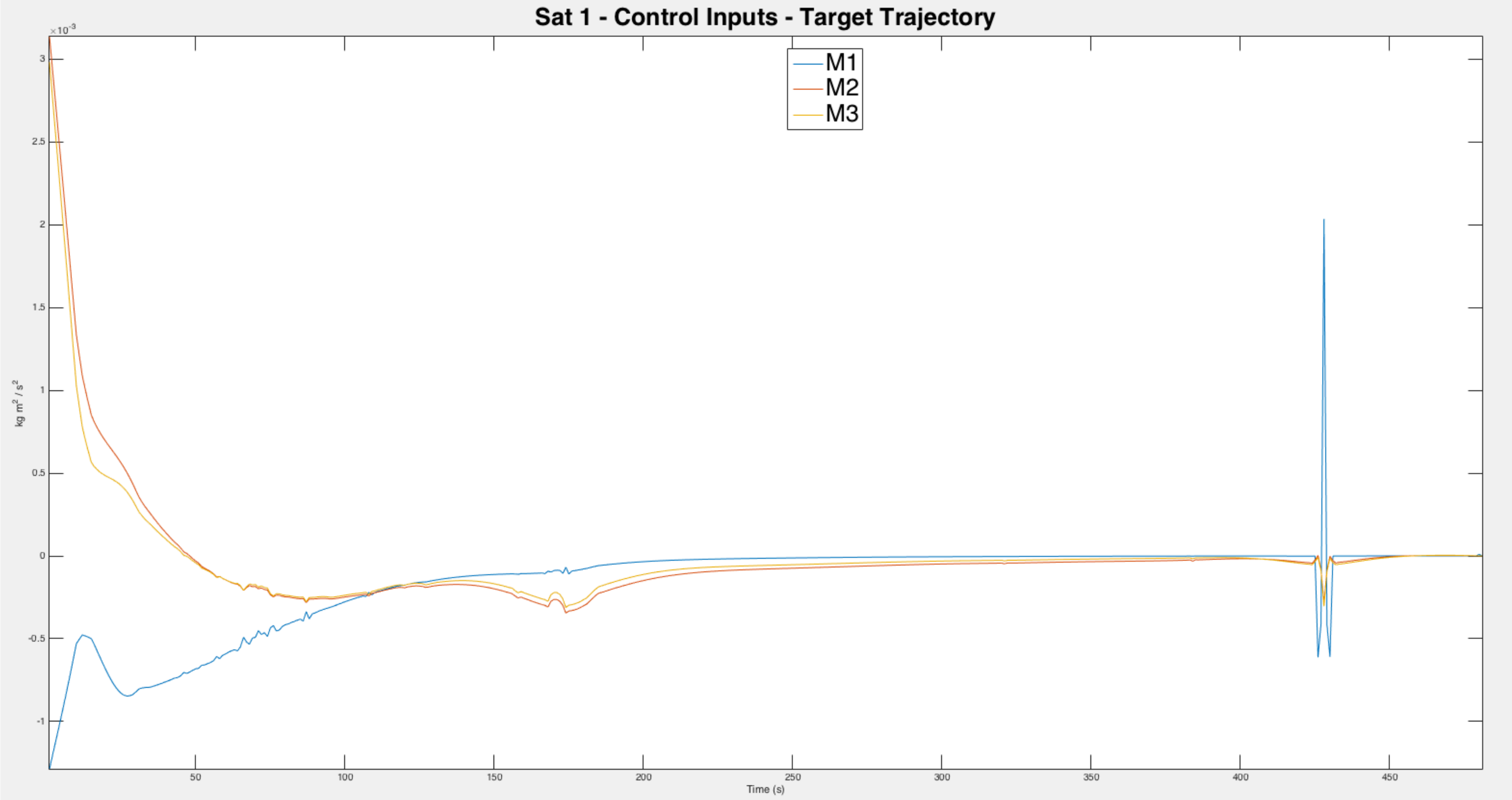
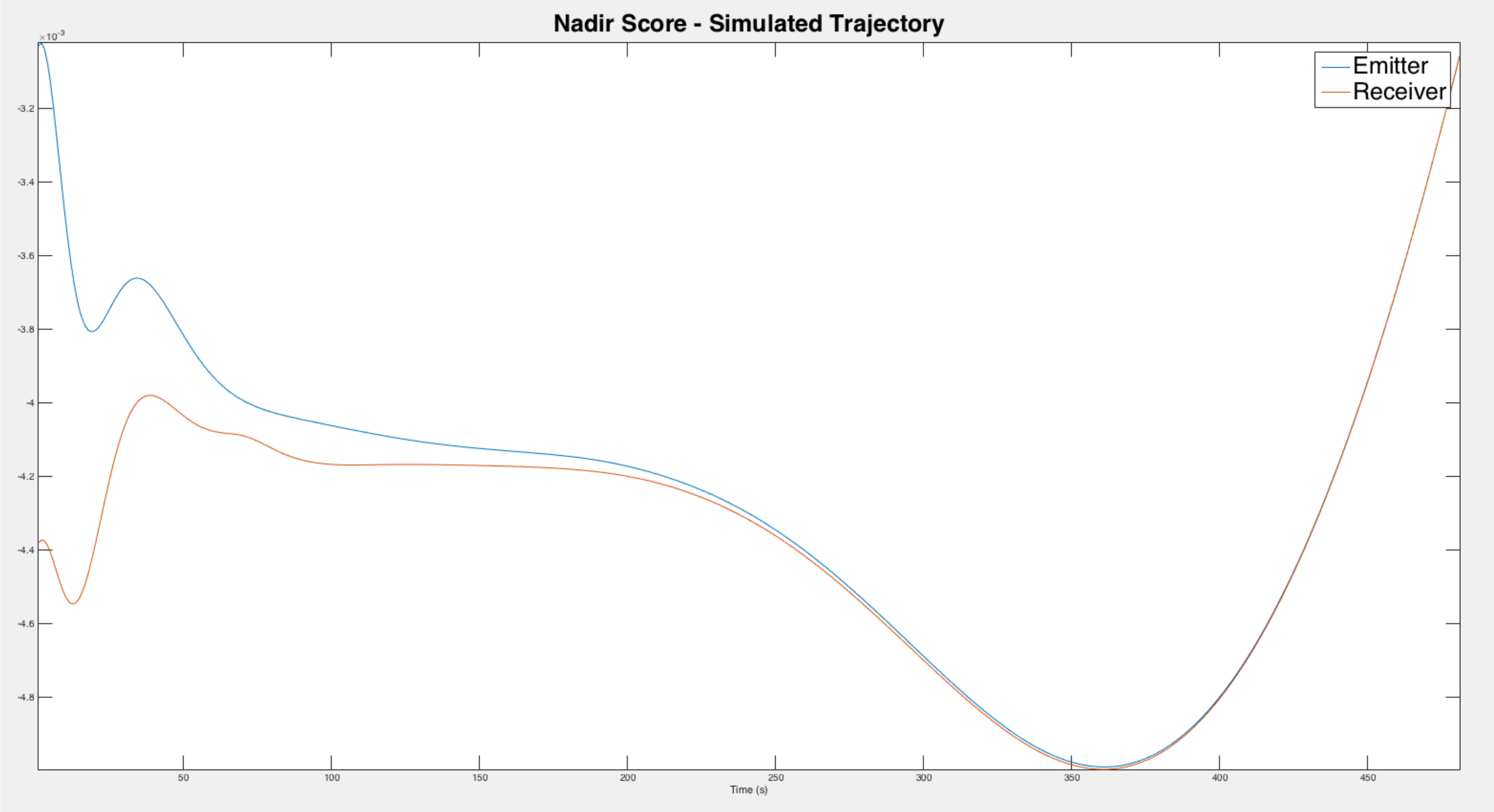
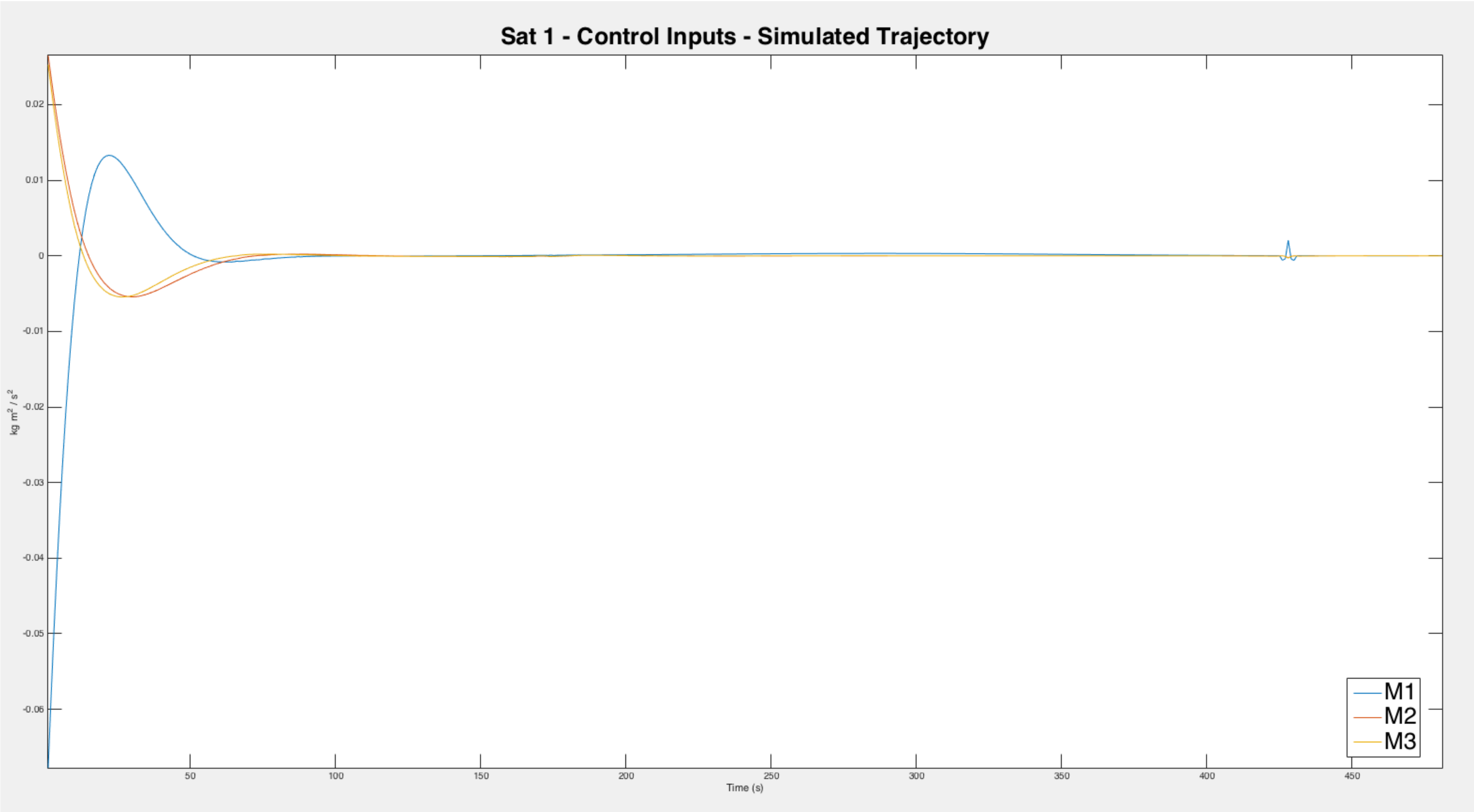
Nadir Constraint



Nadir Constraint

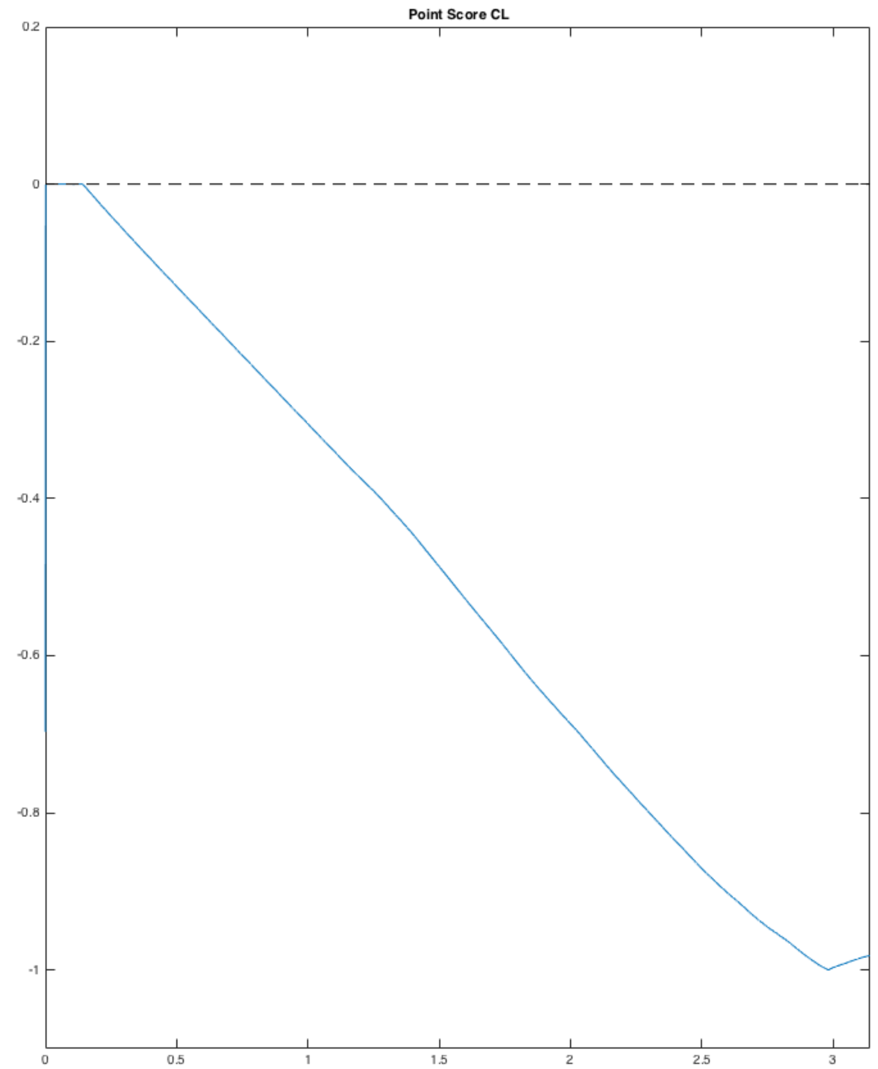


Nadir Constraint



Pointing Constraint?

- Simulated orbit



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